

Cousins

If there were no inbreeding, how many distinct cousins should I have? Let's take a first crack at modeling this problem.

Assume that every person has k children, where k is a nonnegative integer. If that's true, then each of my parents should have $k - 1$ siblings, for a total of $2(k - 1)$ siblings of my parents. Each of those $2(k - 1)$ siblings would produce k offspring, giving a total of $2k(k - 1)$ first cousins for me.

How about second cousins? Again, assume that each of my four grandparents has $k - 1$ siblings. Each of those siblings produces k offspring in my parents' generation. Each of those $4k(k - 1)$ offspring produces k offspring in my generation, for a total of $4k^2(k - 1)$ second cousins.

Continuing this reasoning, it's easy to see (though I flubbed the math a little bit the first time 'round — thanks to Stephanie Goldstein for the catch) that I should have $2^n(k - 1)k^n$ n th cousins.

So then how many cousins should I have — that is, what's the total number of first plus second plus ... n th cousins that I should have? The answer is clear: the total number C_N of my cousins should be

$$\begin{aligned} C_N &= \sum_{n=1}^N 2^n(k - 1)k^n \\ &= (k - 1) \sum_{n=1}^N (2k)^n, \end{aligned}$$

which is a simple geometric series with ratio $2k$. It has a well-known sum:

$$\begin{aligned} C_N &= (k - 1) \left[\sum_{n=0}^N (2k)^n \right] - (2k)^0 \\ &= (k - 1) \left[\frac{1 - (2k)^{N+1}}{1 - 2k} \right] - 1 \\ &= \frac{k - 1}{2k - 1} \left[(2k)^{N+1} - 1 \right] - 1 \end{aligned}$$

With $k = 2$ (two children per person), the total number of cousins exceeds the population of the world somewhere between 16th and 17th cousins. With $k = 3$, we hit that point near my 16th cousins.

But of course there must be a lot of interbreeding; I can't possibly know whether my potential spouse is my 17th cousin. Even if I could, I doubt it

would matter: I only share something like $100/2^{18}\%$, or 0.00038% of my genetic material with my 17th cousins. (That's an offhand guess: I share grandparents with my first cousins, great-grandparents with my second cousins, and so forth.).

The next question is: on average, how many cousins removed am I from every person on earth? Or more generally: what is the probability that I am n cousins removed from any given person, over all n ?